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Modeling of heat transfer in dropwise condensation

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Abstract—A model using the population balance concept is developed to predict the drop size distribution for small drops that grow by direct condensation. The resistances to heat transfer due to the drop (conduction through the drop, vapor–liquid interfacial resistance, drop curvature) and due to the promoter layer and the sweeping effect of falling drops are incorporated into the model and are also included in calculating the heat transfer rate through a single drop. The total heat flux is calculated from the drop size distributions and the heat transfer rate through a single drop. Drop size distribution for large drops that grow by coalescence is obtained from the works of Rose and Glicksman. The work in this paper reveals that to adequately calculate the heat flux, all the resistances to heat transfer due to the drop and the promoter layer have to be included. Considering heat conduction through the drop as the only resistance to heat transfer overestimates the heat flux. The amount of overestimation increases as the temperature difference increases. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

Ever since the discovery of dropwise condensation by Schmidt *et al.* [1] as reported in [2], there has been growing interest in this type of condensation due to its high heat transfer coefficient compared to that in filmwise condensation. For steam condensing dropwise, the heat transfer coefficient could be 20 times higher than for steam condensing filmwise, depending on the temperature difference between the vapor and the condensing surface [3].

Subsequent investigations have dealt mainly with the mechanism of this process, modification of the condensing surface to make the condensation dropwise rather than filmwise and computation of the resulting heat transfer rate.

Since heat transfer is an important aspect of dropwise condensation, various attempts have been made to estimate and/or correlate the heat transfer rate in this process. Fatica and Katz [4] were the first to propose a model to compute the rate of heat transfer by assuming that on a given area all drops are the same size, are uniformly spaced and grow by condensation at their surfaces. In later attempts, different researchers have dealt with the problem of the drop-size distribution in a variety of ways. Le Fevre and Rose [5] assumed a form for the time-averaged distribution which had the correct behavior for the limiting cases of very large and very small drops. Later, Rose and Glicksman [6], based on the results of Westwater and co-workers in which a high magnification cine film was used to observe the sequence of events resulting from sweeping of the condensing surface by departing drops, introduced a universal form for large drops that grow primarily by coalescence with small

drops. In 1981, Rose [7] tried to improve the form by incorporating the resistances to heat transfer due to the drop and various contact angles. Results from the improved formula did not agree well with experimental data. He suggested changing the radius of the smallest viable drop or the maximum drop radius or adding a resistance due to the promoter layer to match the experimental results.

Maa and Wu [8] used the population balance model to derive the drop size distribution of small drops which grow mainly by direct condensation based on the assumption of steady size distribution. They estimated a vapor-side heat transfer coefficient of dropwise condensation by assuming heat conduction through the drop as the only resistance to drop growth.

Later, Maa [9] used the population balance model to derive a drop-size distribution considering both small and large drops on the condensing surface. He solved the resulting equation numerically. The number of nucleation sites was varied so that the result would fit the experimental data.

Tanaka [10–12], based on photographs of a vertical condensing surface, argued that in the so-called steady dropwise condensation, the surface is cleared of condensate periodically by falling drops. Those drops sweep the plate, exposing bare strips, on which transient condensation takes place without delay, until the area is swept clean again. Based on this, he attempted to describe the transient condensation by a set of simultaneous integrodifferential equation derived from statistical and geometrical considerations. The solution to these equations was expressed in terms of four-dimensional parameters, which were adjusted to fit the experimental data.

the promoter layer, and due to the drop shape. The shape of the drop is assumed to be hemispherical. Those resistances are expressed as a temperature difference as shown below.

(1) Drop curvature [5]

$$\Delta T_c = \frac{2T_s\sigma}{H_{fg}\rho} \quad (7)$$

where T_s is the saturation temperature, σ the surface tension, H_{fg} the latent heat of vaporization, ρ the density of the condensate and r the drop radius.

(2) Vapor-liquid interfacial resistance for a hemispherical drop is given by [8]

$$\Delta T_i = \frac{q}{2\pi r^2 h_i} \quad (8)$$

where q is the rate of heat transfer through a drop and h_i is the interfacial heat transfer coefficient defined as

$$h_i = \frac{2\sigma_c}{2 - \sigma_c} \left(\frac{M}{2\pi RT_s} \right)^{1/2} \frac{H_{fg}^2}{T_s v}, \quad (9)$$

where σ_c is the condensation coefficient, it is taken as unity in the calculations which is the case when non-condensable gases are absent, M is the molecular weight, R is the gas constant, and v is the specific volume of the vapor.

(3) Heat conduction through a hemispherical drop is given by [4]

$$\Delta T_d = \frac{qr}{4\pi r^2 k_c} \quad (10)$$

where k_c is the thermal conductivity of the condensate. Henceforth, the above three resistances which are associated with the drop will be called 'drop resistances'.

(4) Heat conduction through the promoter layer

$$\Delta T_p = \frac{qt_p}{4\pi r^2 k_p} \quad (11)$$

where t_p , k_p are the thickness and conductivity of the promoter layer, respectively.

The four temperature differences mentioned above constitute the total temperature difference between the vapor and the condensing surface, which is:

$$\Delta T = \Delta T_c + \Delta T_i + \Delta T_d + \Delta T_p. \quad (12)$$

The minimum radius of the drops that can possibly grow for a given wall subcooling is [14]

$$r_{\min} = \frac{2T_s\sigma}{H_{fg}\rho\Delta T}. \quad (13)$$

Then equation (7) can be expressed as:

$$\Delta T_c = \frac{r_{\min}\Delta T}{r} \quad (14)$$

substituting equations (8), (10), (11) and (14) in equa-

tion (12), and rearranging it, give the heat transfer rate through a single drop as follows

$$q(r) = \frac{4\pi r^2 \left(1 - \frac{r_{\min}}{r}\right) \Delta T}{\frac{r}{k_c} + \frac{2}{h_i} + \frac{t_p}{k_p}}. \quad (15)$$

The rate of heat transfer released from forming a drop can also be expressed as:

$$q(r) = \rho H_{fg} \left(2\pi r^2 \frac{dr}{dt}\right). \quad (16)$$

Equating equations (15) and (16) gives the expression for the drop growth rate as follows:

$$G = \frac{dr}{dt} = \frac{2\Delta T}{\rho H_{fg}} \left(\frac{1 - \frac{r_{\min}}{r}}{\frac{r}{k_c} + \frac{2}{h_i} + \frac{t_p}{k_p}} \right). \quad (17)$$

Since G is a function of r only, equation (6) can be expanded to give the following:

$$-G \frac{dn}{dr} - n \frac{dG}{dr} = \frac{n}{\tau} \quad (18)$$

where

$$\frac{dG}{dr} = \frac{A_1 [r(A_2 r + A_3) - (r - r_{\min})(2A_2 r + A_3)]}{r^2 (A_2 r + A_3)^2} \quad (19)$$

with

$$A_1 = \frac{2\Delta T}{\rho H_{fg}}$$

$$A_2 = \frac{1}{k_c}$$

$$A_3 = \frac{2}{h_i} + \frac{t_p}{k_p}.$$

Substituting G and dG/dr in equation (18) gives the appropriate governing equation. The following is the boundary condition:

$$n(r) = N(r) \quad \text{at} \quad r = r_c \quad (20)$$

where r_c is the radius at the boundary between the coalescence and the non-coalescence size region [8]. This radius is equal to half the mean spacing between the active nucleation sites on the substrate surface. Assuming that the nucleation sites form a square array [6, 8], which gives:

$$r_c = \sqrt{1/4N_s} \quad (21)$$

where N_s is the number of nucleation sites on the condensing surface, $N(r)$ is the well-established drop size-distribution proposed by Rose and Glicksman [6] for large drops which grow by coalescence.

$$N(r) = \frac{(\hat{r}/r)^{2/3}}{3\pi r^2 \hat{r}} \quad (22)$$

also at $r = r_e$ $n(r) = N(r)$ gives

$$\frac{d \ln n(r)}{d \ln r} = \frac{d \ln N(r)}{d \ln r} = -\frac{8}{3}. \quad (23)$$

This relation is used to obtain an expression for τ .

3. RESULTS AND DISCUSSION

3.1. Drop size-distribution

The governing equation as given in equation (18) with the boundary condition (equation 20) is solved analytically. The resulting form is:

$$n(r) = \frac{(\hat{r}/r_e)^{2/3} r (r_e - r_{\min})(A_2 r + A_3)}{3\pi r_e^3 \hat{r} (r - r_{\min})(A_2 r_e + A_3)} \exp(D_1 + D_2) \quad (24)$$

where

$$D_1 = \frac{A_2}{\tau A_1} \left[\frac{r_e^2 - r^2}{2} + r_{\min}(r_e - r) + r_{\min}^2 \ln \left(\frac{r_e - r_{\min}}{r - r_{\min}} \right) \right]$$

$$D_2 = \frac{A_3}{\tau A_1} \left[r_e - r + r_{\min} \ln \left(\frac{r_e - r_{\min}}{r - r_{\min}} \right) \right]$$

with

$$\tau = \frac{3r_e^2 (A_2 r_e + A_3)^2}{A_1 [8A_3 r_e - 14A_2 r_e r_{\min} + 11A_2 r_e^2 - 11A_3 r_{\min}]}$$

and A_1 , A_2 , and A_3 are given in equation (19). The details of the solution are presented in the Appendix.

The drop population density obtained from equations (22) and (24) is plotted in Fig. 1. Up to a drop radius of $0.5 \mu\text{m}$, the rate of decrease in the drop population density is not as steep as the rate for a drop radius greater than $1 \mu\text{m}$. This is because coalescence

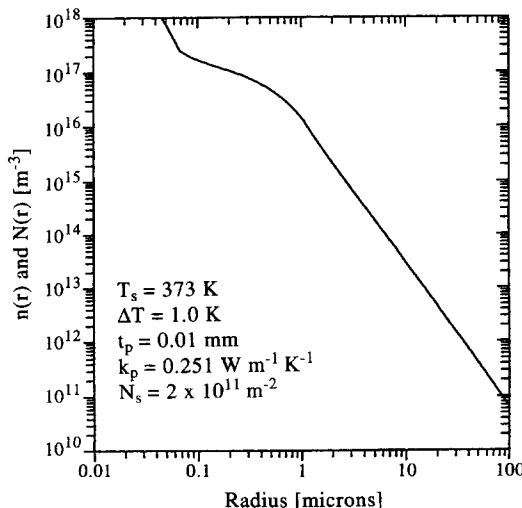


Fig. 1. Population densities of small and large drops as function of drop radius.

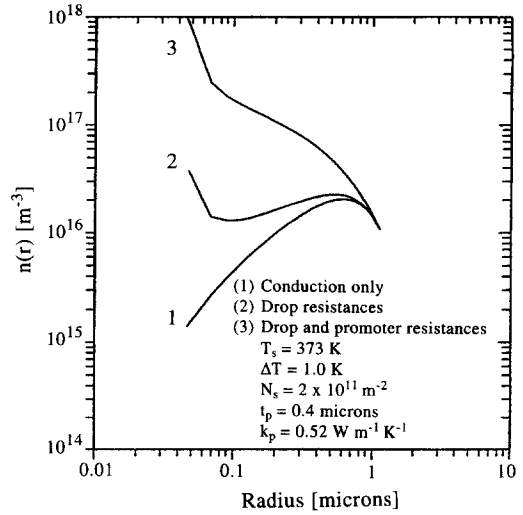


Fig. 2. Population density of small drops as function of drop radius for various resistances.

between drops start taking place when the drop radius reaches 1 or $2 \mu\text{m}$. Figure 2 shows the population density of small drops as a function of radius for three cases. Curve 1 shows $n(r)$ when all the resistances to heat flow due to the drop and the promoter layer are included to obtain the expression for the drop growth rate (G). Curve 2 shows $n(r)$ when the resistances to heat flow due to the drop are included in the expression for G . Curve 3 shows $n(r)$ when only the resistance to heat conduction through the drop body is included to obtain the expression for G . The same procedure described in Section 2 is followed to obtain expression for $n(r)$ in cases 2 and 3. The results are:

for case 2

$$n(r) = \frac{(\hat{r}/r_e)^{2/3} r (r_e - r_{\min})(h_i r + 2k_c)}{3\pi r_e^3 \hat{r} (r - r_{\min})(h_i r_e + 2k_c)} \exp(B_1 + B_2) \quad (25)$$

where

$$B_1 = \frac{2k_c}{A_0 \tau} \left[r_e - r + r_{\min} \ln \left(\frac{r_e - r_{\min}}{r - r_{\min}} \right) \right]$$

$$B_2 = \frac{h_i}{A_0 \tau} \left[\frac{r_e^2 - r^2}{2} + r_{\min}(r_e - r) + r_{\min}^2 \ln \left(\frac{r_e - r_{\min}}{r - r_{\min}} \right) \right]$$

$$A_0 = \frac{2k_c h_i \Delta T}{\rho H_{fg}}$$

$$\tau = \frac{3\rho H_{fg} r_e^2 (h_i r_e + 2k_c)^2}{2\Delta T h_i k_c [11h_i r_e^2 - 14h_i r_{\min} r_e + 16k_c r_e - 22k_c r_{\min}]}$$

For case 3

$$n(r) = \frac{(\hat{r}/r_e)^{2/3}}{3\pi r_e^2 \hat{r}} \left(\frac{r}{r_e} \right) \exp \left(\frac{r_e^2 - r^2}{2a\tau} \right) \quad (26)$$

with

$$a = \frac{2k_c \Delta T}{\rho H_{fg}} \quad \tau = \frac{3r_e^2}{11a}$$

The prediction of the population density of small drops is more realistic when the resistances due to the drop and the promoter layer are included, because $n(r)$ should decrease as the radius of drops increases.

3.2. Heat flux

The heat flux is calculated from the heat transfer through a single drop [equation (15)] and the drop size distributions as given in equations (22) and (24) according to :

$$Q = \int_{r_0}^{r_c} q(r)n(r) dr + \int_{r_c}^{r_{max}} q(r)N(r) dr. \quad (27)$$

The results obtained from this model are shown in Figs. 3–7. Plotting Q vs ΔT gives a straight line as shown in these figures. Figures 3 and 4, which are for low and high pressures, respectively, show the heat

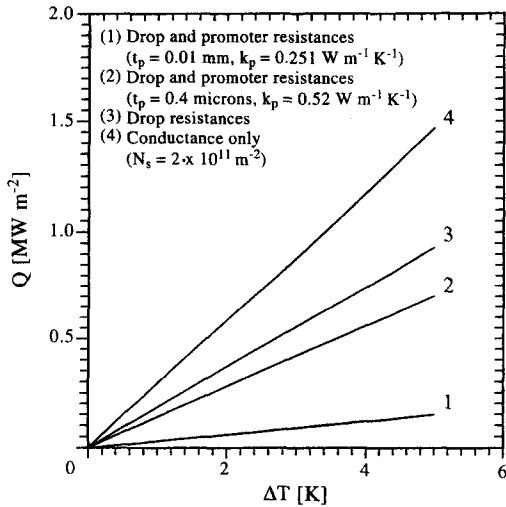


Fig. 3. Predicted heat flux for steam condensing at 306 K.

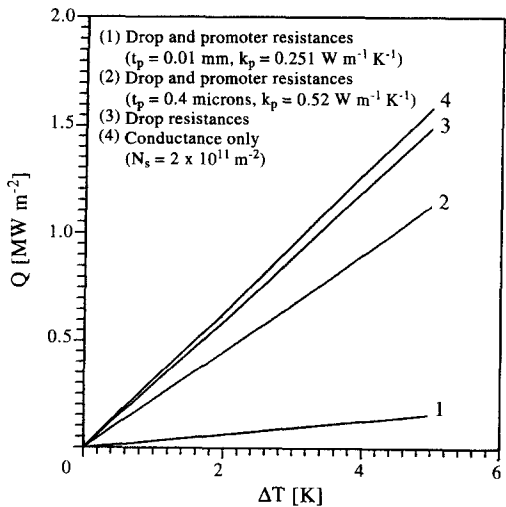


Fig. 4. Predicted heat flux for steam condensing at 373 K.

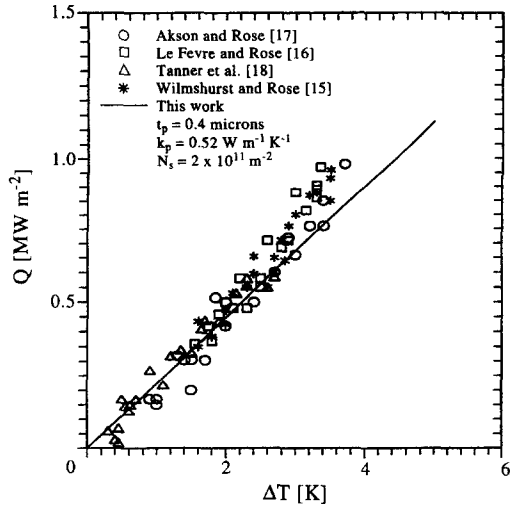


Fig. 5. Predicted heat flux and experimental data from the literature for steam condensing at 373 K.

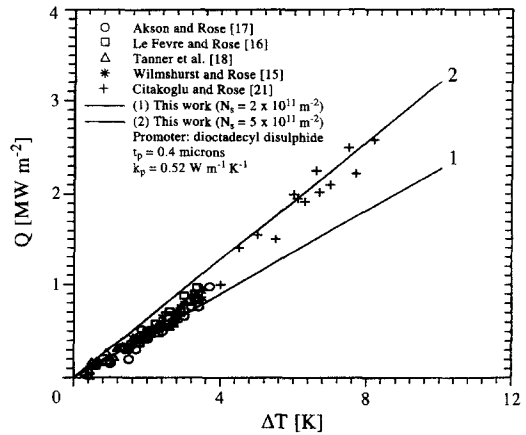


Fig. 6. Predicted heat flux with different number of nucleation sites and experimental data from the literature for steam condensing at 373 K.

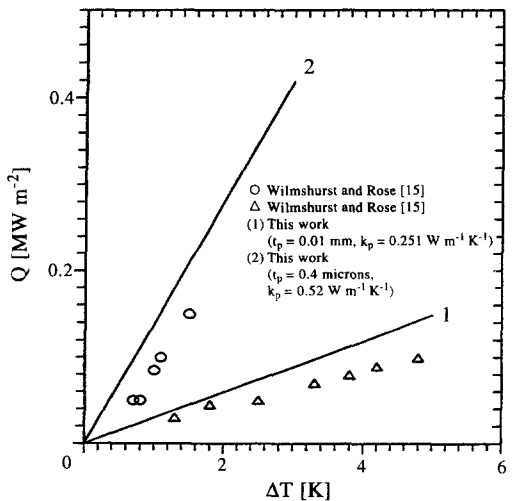


Fig. 7. Predicted heat flux for two different promoters and experimental data from the literature for steam condensing at 306 K.

flux calculated when different resistances to heat flow are incorporated into the model. Curves 1 and 2 are the results of this model for different promoters. The promoters are dioctadecyl disulphide with 0.4 μm thickness and $0.52 \text{ W m}^{-1} \text{ K}^{-1}$ thermal conductivity, and polytetrafluorethylene (PTFE) with 0.01 mm thickness and $0.25 \text{ W m}^{-1} \text{ K}^{-1}$ thermal conductivity. These two promoters were used in several researchers [15–18, 21] experimental work. Their results are compared with the results of the present model in Figs. 5–7 for different saturation temperatures of steam. Curves 3 and 4 in Figs. 2 and 3 are the results obtained when the resistances due to the drop or only conduction through the drop are considered, respectively, with no resistance due to the promoter is included. As can be seen there is a significant difference, mainly as ΔT increases, between these curves and curve 1 or 2. Actually there is a big difference between these curves and curve 1. Figures 3 and 4 show there is a significant difference in heat flux predicted for different promoters, this is in accordance with Le Fevre and Rose's [16] findings. The agreements between the model predictions and the experimental data are good.

It has been reported by McCormick and Westwater [19], and Glicksman and Hunt [20], that the number of nucleation sites increases as ΔT increases. This effect is presented in Fig. 6, which shows a pronounced effect of N_s on the heat flux and is in agreement with the experimental work of Citakoglu and Rose [21].

4. CONCLUSION

In this study, the population balance concept is used to model the drop-size distribution of small drops that grow by direct condensation. In the model, the resistances to heat flow due to the promoter layer and the drop are incorporated. Also, these resistances are incorporated in calculating the heat transfer rate through a single drop.

The predictions show that considering only the resistance to heat conduction through the drop body overestimate the predicted heat flux and underestimate the predicted population density of small drops. Therefore, to adequately calculate the heat flux all the resistances to heat flow due to the drop body and the promoter layer have to be included in the calculations. The predictions show that there are differences in heat fluxes calculated when different promoters are used.

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APPENDIX

To obtain the analytical solution for the number of drops, equation (18) is rearranged to give:

$$G \frac{dn}{dr} + n \frac{dG}{dr} + \frac{n}{\tau} = 0 \quad (\text{A1})$$

letting

$$G_1 = \frac{dG}{dr} + \frac{1}{\tau}$$

where G and dG/dr are given in equations (17) and (19), respectively.

Equation (A1) can be separated to give

$$\frac{dn}{n} = -\frac{G_1}{G} dr. \quad (\text{A2})$$

Integrating equation (A2) gives

$$n(r) = n_0(r) \exp \left[-\int_{r_{\min}}^r \frac{G_1}{G} dr \right]. \quad (\text{A3})$$

Applying the boundary condition, which is $n(r_c) = N(r_c)$ at $r = r_c$ gives :

$$n_0(r) = N(r_c) \exp \left[\int_{r_{\min}}^{r_c} \frac{G_1}{G} dr \right] \quad (\text{A4})$$

Then

$$n(r) = N(r_c) \exp \left[\int_{r_{\min}}^{r_c} \frac{G_1}{G} dr - \int_{r_{\min}}^r \frac{G_1}{G} dr \right] \quad (\text{A5})$$

or

$$n(r) = N(r_c) \exp \left[\int_r^{r_c} \frac{G_1}{G} dr \right]. \quad (\text{A6})$$

Substituting for G and G_1 in equation (A6) and integrating the right-hand side gives the expression for $n(r)$ as :

$$n(r) = N(r_c) \frac{r(r_c - r_{\min})(A_2 r + A_3)}{r_c(r - r_{\min})(A_2 r_c + A_3)} \exp(D_1 + D_2) \quad (\text{A7})$$

where $N(r_c)$, D_1 and D_2 are given in equations (22) and (24), respectively. The sweeping period (τ) is obtained from applying the relation given in equation (23) which gives :

$$\tau = \frac{3r_c^2(A_2 r_c + A_3)^2}{A_1[8A_3 r_c - 14A_2 r_c r_{\min} + 11A_2 r_c^2 - 11A_3 r_{\min}]} \quad (\text{A8})$$